

# From Dipoles to Multipoles : Parametric Solutions to the Inverse Problem in MEG

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## Abstract

In order to reconcile the advantages of the classical dipole model and minimum norm imaging techniques, higher-order parametric source models based on multipolar expansions of the source have been suggested. We evaluate the ability of dipolar and multipolar MEG source estimation frameworks to localize spatially extended patches of cortical activity. The results show the performance of the different models with respect to the area of active cortex and to variable noise conditions and demonstrate the utility of the multipole model.

## 1 Introduction

### 1.1 MEG Inverse Procedures

Reconstruction of the sources underlying magnetoencephalography (MEG) measurements is a severely ill-posed inverse problem typically tackled using either low-dimensional parametric models, such as the equivalent current dipole (ECD), or high-dimensional minimum-norm imaging techniques. The former's inability to properly represent non-focal sources and the latter's tendency to produce over-smoothed solutions underscore the need for an intermediary approach. To this effect, we evaluate a candidate source model based on a truncated multipole expansion: the equivalent current multipole (ECM) [1,2,4].

### 1.2 Dipolar vs. Multipolar Source Models

If the underlying neural sources are assumed to be point-like, the magnetic field measured at the sensor array can be approximated by the magnetic field  $\mathbf{B}_{\text{dip}}$  produced by a current dipole at a location  $\mathbf{l}$  within the conducting medium. The magnetic field outside a spherically symmetric conductor can be expressed as  $\mathbf{B}_{\text{dip}}(\mathbf{r}, \mathbf{l}) = \mathbf{G}_{\text{dip}}(\mathbf{r}, \mathbf{l}) \cdot \mathbf{q}$ , where  $\mathbf{G}_{\text{dip}}$  is the well-known dipole gain matrix and  $\mathbf{q}$  is the dipole moment given by  $\mathbf{q} = \int \mathbf{J}^p(\mathbf{r}') d\mathbf{r}'$  where  $\mathbf{J}^p$  denotes the primary current density. More generally, however, the exact external field produced by an arbitrary source (not necessarily focal) confined to a conducting volume can be expressed by an infinite series expansion such as a Tay-

lor series. The result is a current multipole expansion of the magnetic field [1],

$$\mathbf{B}(\mathbf{r}, \mathbf{l}) = \sum_{n=0}^{\infty} \nabla_{\mathbf{l}}^n \mathbf{G}_{\text{dip}}(\mathbf{r}, \mathbf{l}) \parallel \Omega^n \quad (1)$$

where the double vertical bars denote the  $n$ -fold contractions between the two polyads  $\nabla_{\mathbf{l}}^n$  (the  $n$ th order gradient) and  $\Omega^n$ , the  $n$ th order multipole moment defined as

$$\Omega^n = \frac{1}{n!} \int (\mathbf{r}' - \mathbf{l})^n \mathbf{J}^p(\mathbf{r}') d\mathbf{r}' \quad (2)$$

Hence, all higher-order field components can be derived by successive differentiation of the dipole gain matrix w.r.t. the location  $\mathbf{l}$ . This result holds for arbitrary volume conductors. In practice, a truncated form of equation (1) is used. A zeroth order truncation yields the classical dipole model, whereas truncations at  $n = 1, 2$  or  $3$  lead to multipole expansions up to the quadrupolar, octupolar or hexadecapolar order respectively.

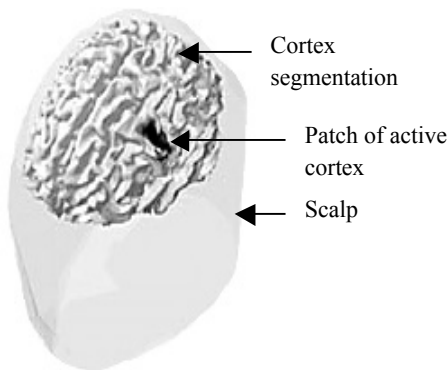
In this paper we evaluate the ability of the current dipole ( $\mathbf{B} \approx \mathbf{B}_{\text{dip}}$ ) and the first-order current multipole ( $\mathbf{B} \approx \mathbf{B}_{\text{dip}} + \mathbf{B}_{\text{quad}}$ ) to localize the centroids of realistic cortical activation patches. When dipolar data are fit with a multipolar model, ambiguities exist between the nonlinear location parameters and the linear moments. Nolte [4] has developed an intermediate model that suppresses this ambiguity in the multipolar moments. Therefore, we investigate two multipole models: an unconstrained rank 7 model (2 dip. + 5 quad. moments) [1] and a restricted rank 4 model (2 dip. + 2 quad. moments) [4]. While the first approach is attractive because it does not systematically assume the

sources to be predominantly dipolar the second is convenient since it automatically removes model redundancies in cases where the dipolarity assumption is correct.

## 2 Localization of Extended Cortical Sources

### 2.1 Realistic Cortical Patches

A tessellated cortex was obtained after segmentation of a high resolution MRI data set using BrainSuite [5]. Realistic cortical activity was then simulated by patches of normally oriented dipoles on the tessellation surface (Fig.1). In order to achieve an appropriate



**Fig. 1** Realistic cortical patch simulations.

sampling of the cortex in terms of patch location, we selected 1019 patch seeds among a total of 10000 vertices. Each seed point was then used to grow an extended patch by including neighbouring vertices until a given surface area was achieved. In this manner, three sets of 1019 patches were created, each with a given average patch size ranging from focal (45 mm<sup>2</sup>), to small (182 mm<sup>2</sup>) to large (430 mm<sup>2</sup>). Furthermore, several noise conditions (SNR) were tested by adding Gaussian white noise to the simulated measurement field of these patches. We used a realistic sensor configuration (151 sensor helmet) and assumed a spherical head model.

### 2.2 Source Localization

For the inverse calculations we used the RAP-MUSIC algorithm [4] as well as a least-squares Levenberg-Marquardt minimization algorithm. The following source models were fitted to the simulation data: the dipole model (ECD), the full (ECM) and restricted (ECM\*) first-order multipole models (Table 1). For each solution, the distance from the solution to the patch's geometric centroid was recorded as an approximate measure of error; however, the notion of a *true centroid* is a matter of definition, and so the term

“error” only denotes a deviation of the solution from the defined patch centroid. The geometric centroid of a distributed source is an intuitively appealing location for its equivalent source model equivalent. We therefore use the distance to the center of mass of a patch's vertices as measure of localization error, denoted *DTC*.

Source Model	Loc	Moments	Rank
ECD	3	2 dip	2
ECM* 1 <sup>st</sup> order, restricted	3	2 dip+2 quad	4
ECM 1 <sup>st</sup> order, full	3	2 dip+5 quad	7

**Table 1** Estimation parameters: location & moments.

## 3 Results

The results in this section were obtained by fitting the three source models of Table 1 to the data sets described in section 2.2. The solutions were *accepted* if the subspace correlation (SC) between the model field and the measurement field exceeds 0.99 [3].

### 3.1 ECD Solutions

For each patch of each data set we ran an ECD fit. The figures 2a, 2b and 2c show the effect of increasing the average patch size on the percentage of successful ECD (white area) localizations. The high percentages and the corresponding average localization error for the accepted ECDs (Table 2) confirm the ability of the ECD to localize most of the source centroids with minimal error. However, the portion of accepted solutions drops significantly as the average patch size increases.

Data set	SC <sub>ECD</sub> >0.99	SC <sub>ECM*</sub> >0.99	SC <sub>ECD</sub> <0.99 but SC <sub>ECM*</sub> >0.99	
	ECD Error	ECM* error	ECM error	ECD error
Focal	1.2	1.3	1.7	9.7
Small	3.4	3.6	5.1	14.3
Large	4.8	5.2	5.1	12.9

**Table 2** Average localization error (DTC) in mm computed for the successful ECD (column 1) and ECM\* models (column 2). In column 3 we show the ECM errors for those sources which could not be localized with >.99 fit using the ECD and, for comparison, the errors that we obtain in fitting the same sources with the ECD. Note that the ECM model is successful in localizing distributed source that are non-dipolar without substantial increases in error.

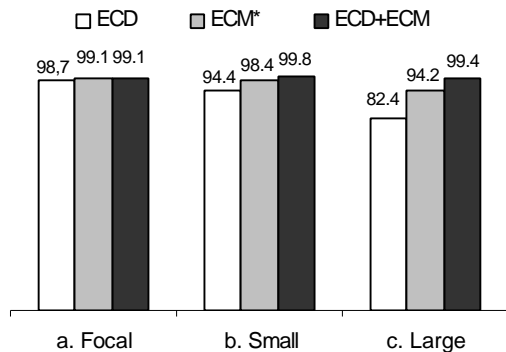
### 3.2 Restricted Multipole Solutions

The restricted ECM\* model was applied to the same data in order to investigate the benefit of the two ad-

ditional quadrupolar terms on localization. Direct comparison between ECM\* and ECD results (Fig.2) show that the former model was able recover more sources, especially as the activated areas increase in size. The average localization error corresponding to these solutions are given in Table 2.

### 3.3 General Multipole Solutions

An alternative approach that uses the unrestricted ECM model is to first run the ECD fit (to avoid subsequent redundancy issues) and keep the solution if the subspace correlation threshold is attained. In case it is not, an ECM fit is run. Similarly, only ECM solutions that achieve the minimum correlation threshold are kept. The rest of the cases are considered to be failed localizations. The accepted ECM solutions are hence a net addition to the successful ECD localizations (ECD%+ECM% (black) in Fig.2). The low dtc errors for the added ECMs (Table 2) show the ability of the model to accurately recover sources that were not localized with the dipole. Furthermore, we also used this hybrid approach to investigate several noise conditions ranging from no noise to 3dB SNR with the data set of large patches (430 mm<sup>2</sup>). The results (Fig 3) and the low dtc errors (no significant deterioration for the kept solutions) demonstrate the robustness of the models w.r.t. additive white noise.

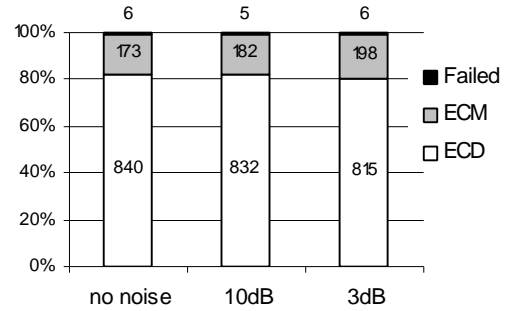


**Fig.2** Source localization success (%) with patch size: a) 45, b) 182 and c) 430 mm<sup>2</sup>. The average localization errors are in Table 2.

## 4 Discussion

Our simulations show that the dipole model does an overall excellent job in modeling cortical activity. Even for the large sources, we observe that the ECD was still able to localize over 80% of the patches within about five mm from the patch's centroid. We thus confirm that the widely-used ECD remains valid for a majority of source configurations.

Moreover, our results demonstrate that even better performances can be achieved by taking advantage of



**Fig.3** Localization success (out of 1019 large patches) in variable noise conditions: a) none, b) 10dB and c) 3dB SNR.

higher-order current multipole expansions. The restricted multipole model (ECM\*) has only two quadrupole moments and yet it leads to a substantial improvement of the field description while maintaining a robust location estimate. Because of its implicit assumption that the measurement field is predominantly dipolar, the restricted multipole can be used instead of the dipole model. For the same reason, however, this model fails if the external field is not strongly dipolar. The parsimonious approach overcomes this restriction by first using the ECD to detect highly dipolar sources and then the ECM to recover sources with strong higher-order components. Future work will include investigating the utility of multipoles for localizing multiple sources and their experimental application.

## 5 Literature

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