Biomagnetism: Applications and Theory,
eds. Harold Weinberg, Gerhard Stroink, Toivo Katil, Pergamon Press, 1985.

THE FORWARD AND INVERSE PROBLEMS IN THE SPHERICAL MODEL

R.J. Ilmoniemi, M.S. Hämäläinen, and J. Knuutila

Low Temperature Laboratory,
Helsinki University of Technology, 02150 Espoo, Finland

ABSTRACT

We analyze the forward problem in the case of the spherical model, introducing simple ways of dealing with the tangential components of the magnetic field. For the inverse problem, we present an alternative to the use of source models, namely the construction of best estimates which are based, in general, both on the results of measurements and on some prior knowledge.

KEYWORDS

Magnetoencephalography; spherical model; forward problem; inverse problem; best estimate; minimum norm estimate.

INTRODUCTION

Traditionally in magnetoencephalography, the magnetic field component normal to the scalp is measured. With single channel magnetometers, this may well be the best approach. However, the designer of multichannel instruments should seriously consider whether the measurement of tangential components at selected points would improve the precision of locating brain events. Another tradition, widely believed to be the only possibility in the determination of sources of biomagnetic fields, is the use of source models. The following discussion attempts to challenge these traditions. For more extensive treatments of pertinent concepts, see, e.g., Cuffin and Cohen (1977) and Tripp (1983).

THE FORWARD PROBLEM IN THE SPHERICAL MODEL

We assume that the conductivity of the head is spherically symmetric about some origin of coordinates: $\sigma(\vec{r}) = \sigma(r)$. Then the radial component of the magnetic field, produced by the current dipole $\vec{Q} = Q\vec{e}_y$ at $\vec{r}_Q = a\vec{e}_z$ (Fig. 1a), can be calculated directly from the dipole expression (Cuffin and Cohen, 1977):

$$B_r(\vec{r}) = \frac{\mu_0}{4\pi} |\vec{r}-a\vec{e}_z|^{-3} \vec{Q}\times(\vec{r}-a\vec{e}_z)\cdot\vec{e}_r = -AG^{-3}\sin\theta\cos\phi , \tag{1}$$

where $A = \mu_0 Q/4\pi a^2$, $G = (1-2\rho\cos\theta+\rho^2)^{1/2}$, and $\rho = r/a$. In the quasistatic approximation, $\nabla\times\vec{B}=0$ outside the head, which leads to $rB_\theta = \int_\infty^r (\partial B_r/\partial\theta)dr$ and $r\sin\theta B_\phi = \int_\infty^r (\partial B_r/\partial\phi)dr$. Inserting the derivatives, obtained from Eq. 1, and integrating, we get the tangential components of the magnetic field:

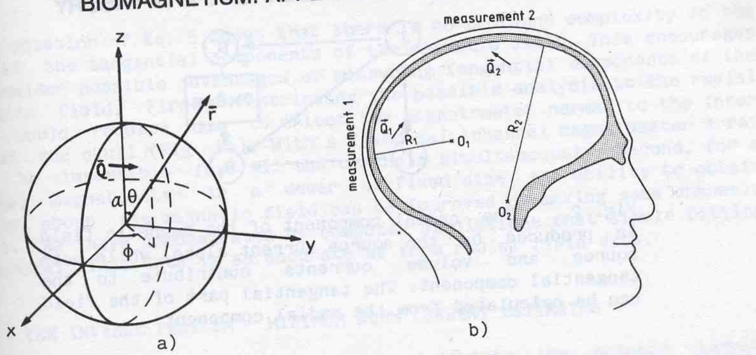

Fig. 1. (a) The coordinate system for the spherical model. (b) The choice of the origin should be based on the local curvature of the skull's inner surface.

$$B_\theta = \frac{-A\cos\phi}{\rho G^3 \sin^2\theta}\left[1+\rho\cos\theta(\rho\cos\theta-1-\cos^2\theta-G^2)+G^3\right] , \tag{2}$$

$$B_\phi = \frac{A\sin\phi}{\rho G\sin^2\theta}(\rho-\cos\theta-G) . \tag{3}$$

To apply the spherical model it suffices that $\sigma(\vec{r})=\sigma(r)$ in the region of appreciable current flow. Figure 1b illustrates this important point: the current flow patterns produced by the dipoles $\vec{Q}_1$ and $\vec{Q}_2$ are influenced mainly by the nearest parts of skull's inner surface with local radii of curvature R_1 and R_2, respectively. Thus, the appropriate origins for the analysis of magnetic field measurements 1 and 2 are O_1 and O_2. The assertion that the choice of origin should be based on the skull's inner surface relies on the assumption that the currents flowing in the poorly conducting skull and in the scalp are not important as far as magnetic fields are concerned.

It is often stated correctly that the effect of volume currents in a spherically symmetric head is zero on the radial field component and nonzero on the tangential part of the field. From this, one is tempted to draw the erroneous conclusion that volume currents complicate the analysis of tangential components. We emphasize that it is not necessary to consider the volume currents explicitly when calculating any of the magnetic field components in the spherical model. The situation is illustrated in Fig. 2. The radial field component B_r is produced solely by the source current distribution J^i, which also gives rise to the volume current J^v; both J^i and J^v contribute to the tangential part of the field B_t. B_t, however, can be determined from B_r, as shown above. How can it be that J^v affects the value of B_t but not B_r while B_t can be calculated from B_r? The answer is simple: the effect of J^v on B_t is independent of any details of conductivity as long as spherical symmetry is maintained. Therefore, although the volume currents are affected by the conductivity profile, their effect on B_t depends only on J^i.

Further insight to the spherical model can be obtained from the diagrammatic equations of Fig. 3. Each circle with a subscript B is a symbol for the magnetic field outside a spherically symmetric head. The arrows denote primary current elements. According to Eq. a, an inversion of current changes the sign of the magnetic field. Eq. b is an assertion of linearity, while Eq. c states that the magnetic field produced by a radial current source is zero. In Eq. d, two zeros are added to the magnetic field of a current element

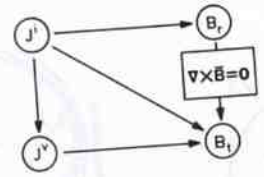

Fig. 2. The radial component of the magnetic field is produced by the source current alone, while both source and volume currents contribute to the tangential component. The tangential part of the field can be calculated from the radial component.

(arbitrary length). Applying Eqs. a and b, we see that the result is an equivalent current loop. The volume current has been replaced by the radial parts of this loop. We notice immediately that the source can roughly be approximated by a magnetic dipole located at the center of mass of the triangle, i.e., two thirds of the distance from the origin towards the current element. The calculation of higher multipole moments from the loop is a simple matter, too. $\vec{B}(\vec{r})$ produced by the loop can be calculated simply from the magnetic scalar potential ϕ_m, defined by $\vec{B} = -\nabla\phi_m$:

$$\phi_m(\vec{r}) = \frac{\mu_0}{4\pi} I \iint d\vec{S}' \cdot (\vec{r}-\vec{r}\,')\,|\vec{r}-\vec{r}\,'|^{-3} , \tag{4}$$

where I is the current circulating in the loop, and $d\vec{S}'$ is a surface element of the triangle. The resulting magnetic field for a short current element, i.e., for a current dipole $\vec{Q}$, located at $\vec{r}_Q$, can be expressed as a linear combination of the vectors $\vec{R} = \vec{r}-\vec{r}_Q$, $\vec{S} = \vec{r}+\vec{r}_Q$, and $\vec{V} = \vec{r}_Q \times \vec{Q}$:

$$\vec{B}(\vec{r}) = \frac{\mu_0}{2\pi} \nabla(tf/d) = \frac{\mu_0}{2\pi d} \left[f\vec{V} + t(g\vec{R} + h\vec{S}) \right] \tag{5}$$

where $d = R^2S^2-(\vec{R}\cdot\vec{S})^2$, $t = \vec{r}\cdot\vec{V}$, $f = R-2r+(\vec{R}\cdot\vec{S})/R$, $h = 2(\vec{R}\cdot\vec{S}-R^2)f/d-(1/r)+1/R$, and $g = 2(\vec{R}\cdot\vec{S}-S^2)f/d-(1/r)+(2/R)-\vec{R}\cdot\vec{S}/R^3$. The effect of volume currents is implicit: the origin must lie at the center of symmetry for Eq. 5 to be correct. In applying Eqs. 1 - 3 to arbitrary dipoles one must compute trigonometric functions and transform between coordinate systems. As these tasks are avoided with Eq. 5, we have been able to speed field calculations by more than a factor of two.

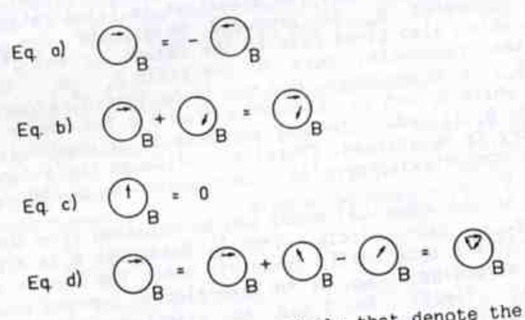

Fig. 3. Equations with symbols that denote the magnetic field outside a spherically symmetric source area.

The vector notation of Eq. 5 shows that there is no profound complexity in the analysis of the tangential components of the magnetic field. This encourages us to consider possible advantages of measuring tangential components of the neuromagnetic field. First, restricting the possible analysis to the radial component would require one to orient the magnetometer normal to the inner surface of the skull (Fig. 1b). With a fixed multichannel magnetometer array this may be impossible for all the channels simultaneously. Second, for a multichannel magnetometer in a dewar of fixed size, the ability to obtain information about the magnetic field can be improved by making some channels tangential. We have demonstrated by computer simulations that dipole fitting from tangential field data is as accurate as from radial field data.

THE INVERSE PROBLEM - MINIMUM NORM CURRENT ESTIMATES

The inverse problem is easy to state: Calculate the primary current distribution in the brain from the magnetic field outside the head. The solution is not unique: an infinity of different current distributions can explain a given magnetic field. Which of the distributions should we choose? Or should the information be expressed in some other form, e.g., by multipole expansions (Katila and Karp, 1983) of the field or of the current? The answer depends on the purpose of the experiment. Multipole coefficients, except for the dipole term in the multipole expansion of the current, seem useful only as tools for mathematical analysis and perhaps for the classification of normal and abnormal heart activity. Our goal is to express the neuromagnetic data in the form of estimates of primary currents in the brain. The neuroscientist is not interested in multipole moments, equivalent current loops, not even in the magnetic field or the field gradient. These are mere tools in the determination of the location and other characteristics of brain activity.

We present the inverse problem in the framework of estimation theory: from measured magnetic field values and some prior knowledge, we want to construct the best estimate for the primary current distribution. It is necessary to define explicitly what is meant by "best". We have studied the application of best estimates to MEG in the hope that, eventually, current density maps will be routinely available for the neuroscientist. We have only taken the first step (Hämäläinen and Ilmoniemi, 1984), demonstrating by simulations and by analyses of measured data that brain activity can be located without using source models.

For our discussion, a few definitions are needed. The lead field of the ith magnetometer, designated by L_i, describes the sensitivity of the magnetometer to primary currents. It is convenient to consider L_i as a vector in an infinite-dimensional function space; the corresponding three-dimensional function, $\vec{L}_i(\vec{r})$, is defined by the equation $B_i = \int \vec{L}_i(\vec{r}) \cdot \vec{J}^p(\vec{r})dV$, where B_i is the output of magnetometer i and $\vec{J}^p(\vec{r})$ is the primary current density at $\vec{r}$. By defining the inner product of L_i and L_j, $(L_i,L_j) = \int \vec{L}_i(\vec{r}) \cdot \vec{L}_j(\vec{r})dV$, we obtain the norm of L_i: $||L_i|| = (L_i,L_i)^{1/2}$. Similarly, the norm of the current distribution J^p is $||J^p|| = (J^p,J^p)^{1/2}$, where $(J^p,J^p) = \int \vec{J}^p(\vec{r}) \cdot \vec{J}^p(\vec{r})dV$. The norm of J^p is a measure for the amplitude of the current distribution.

Of all the current distributions that explain the measured magnetic field, the minimum norm estimate is the one with the smallest norm. If no prior knowledge of the primary current distribution is available, except that it is confined to a known volume of space, e.g. the head, the minimum norm estimate is the best in the sense that the expectation value of $||J^p_{est} - J^p||^2$ is minimized. Here, J^p_{est} is the estimate for the primary current J^p. Fig. 4 shows an example: the left median nerve was stimulated at the wrist, and the magnetic field was recorded successively from the locations indicated by crosses. The rectangles on the left contain isocontour maps for the magnetic field 30 and 150 ms after the stimulus. On the right, corresponding minimum norm estimates are shown 35 mm below the plane defined by the magnetometer locations. It was assumed, for convenience, that the primary current is confined to the rectangle. Of all the current distributions in this rectangle that can explain the measured signals, the one shown has the minimum norm. The measured signals provide no information about the difference between the original current and this estimate. Thus, to improve the estimate, additional information about the source current has to be used. Systematic methods to do this should be the goal of future research on the neuromagnetic inverse problem.

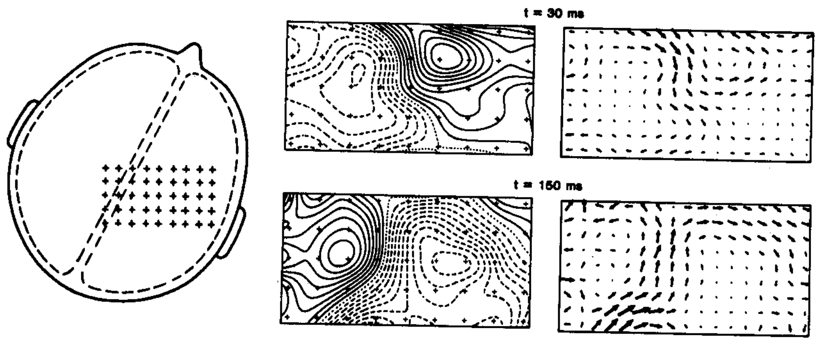

Fig. 4. Isofield maps and minimum norm current estimates. The continuous (positive) and dashed (negative) lines denote flux out of and into the skull, respectively. The crosses show the magnetometer locations, 20 mm apart from each other.

ACKNOWLEDGEMENT

We are grateful to Riitta Hari and Olli Lounasmaa for criticism and encouragement. This work was supported by the Academy of Finland.

REFERENCES

Cuffin, B.N., and D. Cohen (1977). Magnetic fields of a dipole in special volume conductor shapes. IEEE Trans. bio-med. Engng, BME-24, 372-381.

Hämäläinen, M.S., and R.J. Ilmoniemi. Interpreting magnetic fields of the brain - estimates for current distributions, in preparation.

Katila T., and P. Karp (1983). Magnetocardiography: Morphology and multipole presentations. In S.J. Williamson, G.L. Romani, L. Kaufman, and I. Modena (Eds.), Biomagnetism: An Interdisciplinary Approach. Plenum Press, New York. pp. 237-263.

Tripp, J.H. (1983). Physical concepts and mathematical models. In S.J. Williamson, G.L. Romani, L. Kaufman, and I. Modena (Eds.), Biomagnetism: An Interdisciplinary Approach. Plenum Press, New York. pp. 101-139.