Accurate Geometric and Physical Response Modelling for Statistical Image Reconstruction in High Resolution PET

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Abstract

Accurate modeling of the data formation and detection process in PET is essential for optimizing resolution. Here we develop a model in which the following factors are explicitly included: depth dependent geometric sensitivity, photon pair non-collinearity, attenuation, intrinsic detector sensitivity, non-uniform sinogram sampling, crystal penetration and inter-crystal scatter. Statistical reconstruction methods can include these modeling factors in the system matrix that represents the probability of detecting an emission from each image pixel at each detector-pair. We describe a method for computing these factors using a combination of calibration measurements, geometric modeling and Monte Carlo computation. By assuming that blurring effects and depth dependent sensitivities are separable, we are able to exploit rotational symmetries with respect to the sinogram. This results in substantial savings in both storage requirements and computational costs. Using phantom data we show that this system model can produce higher resolution near the center of the field of view, at a given SNR, than both simpler geometric models and reconstructions using filtered backprojection. We also show, using an off-centered phantom, that larger improvements in resolution occur towards the edge of the field of view due to the explicit modeling of crystal penetration effects.

1 Introduction

Quantitative PET image reconstruction using statistical techniques requires a system model that represents the probability of detecting an emission from each image pixel at each detector-pair. By accurately modeling these probabilities, we can minimize spatial distortions due to simplifying assumptions and can maximize resolution recovery. The typical model for true coincidences used in iterative PET image reconstruction is based on a simple geometric model in which the contributions of each pixel to each sinogram element are assumed proportional to the area of intersection of the pixel and the strip joining the two detectors [10]. The model is then modified using calibration measurements to also include attenuation and detector sensitivity factors. Factors that are often ignored in these simplified system models include depth dependent geometric sensitivities, photon pair non-collinearity, non-uniform sampling, crystal penetration and inter-crystal scatter. These factors are becoming increasingly important with the reduced crystal size and potential for much higher resolution of the newer generation of scanners. We describe below how they can be incorporated in an efficient manner.

We use the standard Poisson model for PET data in which the intensities in each pixel are related to the mean of the true coincidences through a linear transform defined by a matrix P. In the following, randoms and scatter are not explicitly considered; the model can be extended to include these as we describe in [8]. Our approach to modeling P is an extension of the factored matrix method described in [9] and [8]. We decouple the depth dependent geometric sensitivities from the blurring effects. In this way the geometric projection matrix has the same degree of sparseness as the simpler system models. The blurring effects due to non-collinearity and inter-crystal scatter and penetration, are included as a local, spatially variant blur in the sinogram space. While spatially variant sinogram blurring models have been considered previously [6] [7], these models were used for sinogram restoration in conjunction with filtered backprojection rather than as part of a statistical reconstruction method. Carson et al [1] model these blurring effects in a similar manner to that described here, except that they restrict blurring to the radial direction in the sinogram and apparently do not model the depth dependent geometric sensitivities.

Our purpose in this paper is to develop the factored matrix representation of the system and show that using this system matrix within a regularized statistical image reconstruction framework can produce resolution recovery. By using the factored matrix approach we gain substantial savings in both storage and computational requirements - roughly a factor of 3 for the brain scan example shown in Section 3.2. We note however, that to realize this saving, it is necessary to consider all data and all pixels at each iteration. The factored matrix approach is therefore most compatible with the original EM algorithm and gradient based update methods such as the conjugate gradient approach in [8]. Coordinate wise methods are inappropriate since only one pixel is updated per iteration. Similarly the ordered subsets approach [5] will not realize the potential savings since not all data are used at each iteration.

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2 Factored System Model

The true coincident data are modeled as Poisson with mean given by \( \hat{c}_i(\lambda) = \sum_j p_{ij} \lambda_j \) where \( \lambda \) denotes the source image. The elements \( p_{ij} \) of the matrix \( P \) denote the probability of detecting an emission from pixel site \( j \) at detector pair \( i \). Let \( M \) be the number of detector pairs and \( N^2 \) the number of pixels in the image. Then \( P \) is an \( M \times N^2 \) matrix which we factor as follows:

\[
P = P(\text{det.sens}) P(\text{atten}) P(\text{det.blur}) P(\text{geom})
\]

The factor \( P(\text{geom}) \) is an \( M \times N^2 \) matrix with element \((i,j)\) equal to the probability that a photon pair produced in pixel \( j \) reaches the front faces of the detector pair \( i \) in the absence of an attenuating medium and assuming perfect photon-pair colinearity. This coincidence response is well modeled as a trapezoidal function with depth dependent parameters [4]. This response changes slowly from triangular in the center to square at the detector surfaces, due to depth dependent changes in the solid angle subtended by the pixel at the two detectors. For pixel sizes smaller than the detector width, each pixel will contribute to at most two sinogram elements per angle of view and consequently this matrix is very sparse with approximately \( P \times N^2 \) non-zero values, where \( P \) is the number of angles of view.

In principal, uncertainties in the angular separation of the photon pair should also be included in \( P(\text{geom}) \). However, this will lead to an increase in the number of sinogram elements to which each pixel contributes and hence a reduction in the sparseness of the matrix. To a reasonable approximation, we can assume that the non-colinearity effect is depth-independent and lump this effect with blurring within the detector in the matrix \( P(\text{det.blur}) \).

The block detector structure limits our ability to detect and reject photons that scatter within the detector block. This is due to the use of a small number of PMTs to detect photons from a larger number of crystals. Positioning using Anger logic results in mispositioning of events due to detection of multiple interactions of the scattered photons from a larger number of crystals. P ositioning This is due to the use of a small number of PMTs to de-

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The blurring factors are computed using a Monte Carlo code. Space limitations preclude a full description of the model. To summarize, we compute the blurring associated with each of the \( (1 + L/2) \) detector pairs by tracing individual photons drawn uniformly from an in-plane line source oriented normal to, and at the center of, the line of response, and with length equal to the width of the line of response. Only photon pairs that would strike the detector pair under consideration, in the absence of the blurring factors, are allowed to contribute to the sinogram blurring kernel. Statistical modeling of non-colinearity, crystal penetration and inter-crystal scatter is used to produce the blurring of the sinogram element under consideration into the neighboring elements. The code is run for sufficient time to generate a high SNR estimate of the blurring kernel. We then truncate the kernel at 1% of its maximum value to produce an appropriate neighborhood for the blurring kernel - this increases towards the edge of the field of view due to increased crystal penetration effects. The truncated kernel is then normalized so that it sums to unity since geometric and intrinsic sensitivities are modeled in other factors. Comparing \( P(\text{det.blur}) \) for line sources at various depths we found no more than 4% RMS difference, verifying the effective depth independence of the factors modeled in this matrix.

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The attenuation matrix \( P(\text{atten}) \) is a diagonal \( M \times M \) matrix that contains the attenuation correction factors for each detector pair and can be computed in the usual way from the ratio of a blank to transmission scan. Alternatively, these factors can be computed by forward projecting a reconstructed attenuation image. If a statistically based reconstruction method is used, as described in [8], the reprojec-

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The final component of the factored matrix is \( P(\text{det.sens}) \) which is again a diagonal \( M \times M \) matrix and contains the detector efficiencies. The efficiency of detector pair \((k,l)\) can be expressed as [2]

\[
E_{k,l} = f_{k,l} \ast g_{k,l} \ast \eta_{k,l} \ast d_{k,l}
\]

where \( f_{k,l} \) represents the radially varying geometric efficiencies due to the ring structure of the detectors; \( g_{k,l} \) represents detector-pair geometric efficiencies due to non-equal detector surface areas within the block; \( \eta_{k,l} = \epsilon_k \ast \eta_l \) is the
product of the intrinsic detector efficiencies, which are due to crystal imperfections, light guide variations, differences in PMT gains and variations in the electronics used to detect PMT signals; finally, \(d_{k,l}\) represents the non-uniform loss of counts due to deadtime. In our model we use the deadtime correction factors computed based on the singles rates of the detector blocks [2]. The factor \(f_{ij}\) is modeled in \(P(\text{geom})\); the remaining factors must be included in \(P(\text{det.sens})\) which is an \(M \times M\) diagonal matrix with diagonal element \((i,i)\) equal to \(g_{k,i} * k_{k,i} * d_{k,l}\) where \((k,l)\) are the detector pair associated with sinogram element \(i\).

3 Experimental Evaluations

3.1 Line Source Measurements

A pair of 1.5 mm diameter line sources oriented axially (length 5 cm) were scanned in a Siemens/CTI ECAT EXACT HR+ scanner. In Fig. 2(a) we compare the sinogram profile measured using the line sources with that predicted using the factored system model described above. Note the good agreement between the two and that the asymmetry and peak shift due to crystal penetration is successfully followed. In comparison, the geometric-only model \((P(\text{det.blur})\) not included) produces a poor fit to the sinogram as shown in Fig. 2(b). The small errors in the tails for the full model are probably due to factors that are not modeled such as septal penetration and axial effects.

3.2 Phantom Experiments

We scanned the hot-spot Derenzo phantom (20 cm diameter) in the ECAT EXACT HR+, both at the center of the field of view, and also with the center off-set by a radial distance of 16.40 cm. Reconstructions of the phantom are shown in Fig. 3 and Fig. 6. These results are for high count data (7 million counts per plane). The quadratically regularized weighted least squares (WLS) method with resolution matching [3] was used rather than the MAP method in [8] to avoid realizing artificially high resolutions due to the positivity constraint used in the MAP method. Profiles through FBP and WLS reconstructions are shown in Fig. 4. To evaluate performance, we reconstructed the image for several values of the smoothing parameter and compared the background noise level from the phantom study with the resolution at the center of field of view as measured using the line source data. The result is shown in Fig. 5 in comparison to that for FBP with different cut-off frequencies. This result shows that we can achieve resolution recovery using the full model without unacceptable increases in noise. Note that the 4mm FWHM resolution using the full model has approximately the same noise level as that for FBP with a resolution of 5.25mm FWHM.

In Fig. 6 we show reconstructions using FBP and WLS for the phantom placed 16.40 cm from the center of the field of view. In this case, the substantial resolution improvement due to modeling of detector penetration is clear. Finally, in Fig. 7 we show the potential for resolution recovery in a normal, high count, FDG brain study by comparing WLS with the full model to ramp-filtered FBP.

4 Conclusions

We have described a procedure for generating an accurate factored system matrix for 2D PET and have demonstrated improvements in resolution throughout the field of view using phantom and human data. Using the factored matrix approach and reconstruction algorithms in which the full image is updated at each iteration, this full model can be included at approximately the same cost as one based on the simpler geometric model.

References


Figure 1: Illustration of the rotational invariance, with respect to the detectors, of the 2D sinogram blurring kernels.

Figure 2: Profiles through the measured (dots) and calculated (solid line) sinograms for two line sources for the full system model (top) and geometric model only (bottom). Sinogram components 100 and 8 correspond approximately to lines of response at the center and 20 cm from the center of the field of view, respectively.

Figure 3: Reconstructions of the Derenzo phantom, centered in the field of view, using ramp filtered FBP (top), regularized WLS with the full model (center), and regularized WLS with geometric model only (bottom). FBP and WLS images are reconstructed at 4.65 mm and 4.00 mm FWHM resolution, respectively.
Figure 4: Profiles (108th column of $220 \times 220$ images) through the reconstructions of the centered Derenzo phantom: FBP (solid line) and WLS with full model (dashed line).

Figure 5: Plot of the background noise variance in the Derenzo phantom vs. resolution as measured using the centered line source for different degrees of smoothing using: FBP (squares), WLS with geometric model only (open circles), and WLS with full model (closed circles).

Figure 6: Reconstructions of the Derenzo phantom with the center placed 16.40 cm from the center of the field of view using FBP (left) and WLS with the full model (right).

Figure 7: Reconstructions of a human FDG brain scan (7 million counts) using FBP (top) and WLS with the full model (bottom) at 4.65 mm and 4.00 mm FWHM resolution, respectively.