

# MATRIX KERNELS FOR MEG AND EEG SOURCE LOCALIZATION AND IMAGING

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## ABSTRACT

The most widely used model for electroencephalography (EEG) and magnetoencephalography (MEG) assumes a quasi-static approximation of Maxwell's equations and a piecewise homogeneous conductor model. Both models contain an incremental field element that linearly relates an incremental source element (current dipole) to the field or voltage at a distant point. The explicit form of the field element is dependent on the head modeling assumptions and sensor configuration. Proper characterization of this incremental element is crucial to the inverse problem. The field element can be partitioned into the product of a vector dependent on sensor characteristics and a matrix kernel dependent only on head modeling assumptions. We present here the matrix kernels for the general boundary element model (BEM) and for MEG spherical models. We show how these kernels are easily interchanged in a linear algebraic framework that includes sensor specifics such as orientation and gradiometer configuration. We then describe how this kernel is easily applied to "gain" or "transfer" matrices used in multiple dipole and source imaging models.

## 1. INTRODUCTION

In [1] we investigated several methods of image reconstruction, essentially using a discrete form of the magnetic lead field. In [2], we assumed the source to be a small set of point sources, i.e., current dipoles at unknown locations. This approach focuses the processing on the magnetic forward field. We also showed how the "fixed," "rotating," and "moving" multiple dipole models could all be cast in the same linear algebraic framework.

These previous models emphasized the *source* model. For simplicity in those presentations, we assumed a very simple *head* model and *sensor* configuration. In this paper, we present the explicit matrix kernels for different head models, then describe how they can be interchanged in a common set of equations describing the sensor characteristics. The general BEM head model results are applicable to both EEG and MEG; for brevity, the model for a spherical head is presented for the MEG case only.

## 2. PRIMARY AND VOLUME CURRENTS

For the biological signals of interest in MEG and EEG, the time-derivatives of the associated electric and magnetic fields are sufficiently small such that they can be ignored in Maxwell's equations. See Equations (3)–(9) of ([3]) for a recent discussion and ([4]) for more details. The static magnetic field equations are  $\nabla \times \mathbf{B}(\mathbf{r}) = \mu_0 \mathbf{J}(\mathbf{r})$  and  $\nabla \cdot \mathbf{B}(\mathbf{r}) = 0$ , i.e., the curl of the magnetic field at location  $\mathbf{r}$  is equal to the current density (times  $\mu_0$ ), and the divergence of the magnetic field is zero.

We are interested in the current density  $\mathbf{J}$  in a closed volume of finite conductivities. Outside this volume the conductivity and current density are zero. We divide the current into two components, passive and primary. We define as passive those currents that are a

result of the macroscopic electric field in the conducting medium of the volume,  $\mathbf{J}^v = \sigma \mathbf{E}$ . All other currents are considered primary,  $\mathbf{J}^p$ . The division of the current as  $\mathbf{J}^p$  and  $\mathbf{J}^v$  is to emphasize that neural activity gives rise to primary currents that then flow passively throughout the rest of the conducting medium. Our problem is to locate these primary currents and hence the sources of brain activity.

Substituting our interpretation of  $\mathbf{J}$  into the equation for the quasi-static magnetic field yields (c.f., [3])

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int_G \left( \mathbf{J}^p(\mathbf{r}') - \sigma(\mathbf{r}') \nabla V(\mathbf{r}') \right) \times \frac{(\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} d\mathbf{v}'. \quad (1)$$

with the integration carried out over a closed volume  $G$ .

The typically accepted brain model assumes piecewise homogeneous conductive regions (we will consider only isotropic conductivities here). The gradient of the conductivity is therefore zero except at the surfaces between regions; therefore, the volume integrals can be reworked into surface integrals. These surface integrals allow possibly simpler solutions and smaller sets of numerical equations, as well as some simplifications for specialized cases.

We assume our volume can be divided into  $M+1$  different regions with conductivities  $\sigma_i$ , including the nonconducting region outside of the head. These regions are separated from adjacent regions by a total of  $m \geq M$  surfaces  $S_i$ . Through a sequence of steps using vector identities, we can rewrite (1) as ([3], [4], [5])

$$\mathbf{B}(\mathbf{r}) = \mathbf{B}_0(\mathbf{r}) - \frac{\mu_0}{4\pi} \sum_{i=1}^m \left( \sigma_i^- - \sigma_i^+ \right) \left( \int_{S_i} V(\mathbf{r}') \mathbf{n}_i(\mathbf{r}') \times \frac{(\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} d\mathbf{a}' \right) \quad (2)$$

where  $\mathbf{n}_i(\mathbf{r})$  is the "outward" directed unit vector normal to the  $i$ th surface, and the "+" ("−") superscript indicates the conductivity outside (inside) the  $i$ th surface.  $\mathbf{B}_0(\mathbf{r})$  is the "primary current model":

$$\mathbf{B}_0(\mathbf{r}) = \frac{\mu_0}{4\pi} \int_G \frac{\mathbf{J}^p(\mathbf{r}') \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} d\mathbf{v}', \quad (3)$$

which is the magnetic field observed at  $\mathbf{r}$  due to the primary current only. We defer the explicit details to ([3], [4], [5]). Using (1) Green's second identity, (2) a continuous current density across each interface, (3) the divergence free nature of the total current, and (4) limits as a point in the volume approaches a surface, we can obtain a surface integral equation for  $V(\mathbf{r})$  for  $\mathbf{r}$  on the  $j$ th surface (with elegant symmetry to the magnetic equation),

$$\begin{aligned} & \frac{(\sigma_j^- + \sigma_j^+)}{2} V(\mathbf{r}) = \\ & \sigma_0 V_0(\mathbf{r}) - \frac{1}{4\pi} \sum_{i=1}^m \left( \sigma_i^- - \sigma_i^+ \right) \int_{S_i} V(\mathbf{r}') \mathbf{n}_i(\mathbf{r}') \cdot \frac{(\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} d\mathbf{a}' \end{aligned} \quad (4)$$

$V_0(\mathbf{r})$  is the solution for the infinite homogeneous medium due to the primary current  $\mathbf{J}^p$ , and  $\sigma_0$  is the normalizing unit conductivity needed for correct dimensions ([3],[4], [5]).

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$$V_o(r) = -\frac{1}{4\pi\sigma_0} \int_G \mathbf{J}^p(r') \cdot \frac{(r-r')}{|r-r'|^3} dv'. \quad (5)$$

Equations (2) and (4) therefore form our general set of equations for solving the forward problem for both voltages and magnetic fields.

### 2.1 Point Dipole

For the case of  $\mathbf{J}^p(r) = q\delta(r-r_q)$ , i.e., the current dipole  $q$  located at  $r_q$ , then  $B_o(r)$  and  $V_o(r)$  can be simplified as (c.f., [3], [4], [5], [6])

$$B_o(r) = \frac{\mu_0 q \times (r-r_q)}{4\pi |r-r_q|^3}, \quad V_o(r) = \frac{1}{4\pi\sigma_0} \frac{q \cdot (r-r_q)}{|r-r_q|^3} \quad (6)$$

### 2.2 Spherically Symmetric Conductor

In the spherically symmetric conductor case,  $n(r') = r'/|r'|$  for all  $r'$  on all surfaces. If we examine just the radial component of the field at  $r$ ,  $B_r = \mathbf{B}(r) \cdot r/|r|$ , then the contribution due to the passive currents in (2) can be shown to vanish. Thus  $B_r$  is simply calculated with the well-known *primary current model*,

$$B_r = B_o(r) \cdot r = \frac{\mu_0 q \times (r-r_q) \cdot r}{4\pi |r-r_q|^3 |r|} = \frac{\mu_0 r_q \times q \cdot r}{4\pi |r-r_q|^3 |r|} \quad (7)$$

Since no currents exist outside the head, the radial magnetic field  $B_r$  can be used to derive the scalar magnetic potential  $U(r)$ , and hence the full magnetic field is derived as the gradient of this scalar. Sarvas has explicitly stated the formula for  $\mathbf{B}(r) = -\mu_0 \nabla U(r)$  outside the spherical conductor in Cartesian coordinates ([5], repeated [3] Eq. (34)),

$$\mathbf{B}(r) = \frac{\mu_0}{4\pi F^2} (Fq \times r_q - (q \times r_q \cdot r) \nabla F) \quad (8)$$

where  $F$  is a scalar function,  $\nabla F$  is a vector function, and both are functions of  $r$  and  $r_q$ ,

$$F = F(r, r_q) = a(ra + r^2 - (r_q \cdot r)) \quad (9)$$

$$\nabla F = \nabla F(r, r_q) =$$

$$\left( r^{-1} a^2 + a^{-1} (a \cdot r) + 2a + 2r \right) r - \left( a + 2r + a^{-1} (a \cdot r) \right) r_q \quad (10)$$

and  $a = r - r_q$ ,  $a = |a|$ , and  $r = |r|$ .

The equation for the voltage potential on the surface of concentric spherical shells is not as compactly presented as for the MEG case. The EEG formula using similar notation and references to previous derivations can be found in [7].

### 2.3 Boundary Element Method

The review in [3] of the boundary element method and the references therein provide a good perspective on the approaches and specifics of modeling more realistically shaped heads. Here we can only give a brief overview to motivate the gain matrix calculations. In the arbitrarily shaped conductor, we compute the surface integrals of the voltages using the boundary element method. We can tessellate each surface  $S_i$  into  $d_i$  suitable triangles,  $\Delta_k^i$ ,  $k = 1, \dots, d_i$ . The area of each triangle is denoted  $\mu_k^i$ . If we assume that the voltage is constant across each triangle,  $V(r) = V_k^i$ , then integrate (4) over one triangle, we can develop a set of linear equations,

$$V^i = \sum_{j=1}^{d_i} H^{ij} V^j + g^i \quad i = 1, \dots, m \quad (11)$$

See [3], [6], for detailed definitions of  $H$  and  $g$ . The term  $g$  is linearly related to the primary voltage  $V_o$ , and the dipole  $q$  can be factored out, such that  $g = G_o q$  over the system of triangles.  $H$  can be

computed once the conductor geometry is known, independently of any source or sensor location considerations. Since voltage potentials are defined up to an additive constant,  $H$  must be "deflated," ([3], [8]) to  $\tilde{H}$ . This deflation allows us to now uniquely solve the system of  $N$  equations,

$$(I - \tilde{H}) V = g = G_o q \quad (12)$$

where  $I$  is an  $N \times N$  identity matrix. After solving for  $V$ , we have essentially solved the EEG problem for all surface elements in the head.

The magnetic field is found from a discrete form of (2), where we can again approximate the integration using the centroid of the triangles ([6]), yielding

$$\mathbf{B}(r) = B_o(r) + A V = B_o(r) + A (I - \tilde{H})^{-1} g \quad (13)$$

where matrix  $A$  represents the linear transfer function that relates  $V$  to the contribution that the passive currents add to the magnetic field at point  $r$ . The matrix  $A$  is a function of the sensor location and the head geometry and can therefore be computed without regard to the source currents  $\mathbf{J}^p$ . The right hand expression in (13) has the advantage of not explicitly calculating the EEG potentials, an approach that can greatly reduce the numerical calculation cost.

As we illustrate below,  $B_o$  can also be expressed as a kernel matrix times the dipole,  $B_o = K_o q$ . Combining these terms allows us to express (13) for the dipole case as

$$\mathbf{B}(r) = [K_o + A (I - \tilde{H})^{-1} G_o] q \quad (14)$$

## 3. MATRIX KERNELS

Each of the above models, (6), (8), and (14) for MEG, and (12) for EEG, represents a linear relationship between the dipole moment  $q$  and the field  $B$  (or measurement  $V$ ). In this section, we represent each model as  $B$  (or  $V$ ) =  $K(r, r_q) q$ , where  $K$  is a matrix kernel. Although (6), (12), and (14) have an obvious form for  $K$ , (8) does not.

To simplify the algebraic manipulation of the cross-product, we convert the operation to the product of a matrix and a vector and explicitly state all vectors in their Cartesian forms,

$$a \times b = C_a b = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} \quad (15)$$

Substituting into (8) and manipulating the triple scalar product, we find the expression for the spherical model,

$$B_s(r) = \frac{\mu_0}{4\pi F^2} \left( (-F) (C_{r_q} q) + (\nabla F) (r^T C_{r_q} q) \right) \quad (16)$$

$$= \left[ \frac{\mu_0}{4\pi} \frac{[\nabla F r^T - F I]}{F^2} C_{r_q} \right] q = K_s(r, r_q) q \quad (17)$$

The kernel for the spherical EEG case can be extracted from the concentric shell model; see [7] and its references.

The other kernels are easily formed. The widely used primary current model is

$$B_o(r) = \left[ \frac{\mu_0}{4\pi} \frac{C_{r_q} - C_r}{|r-r_q|^3} \right] q = K_o(r, r_q) q \quad (18)$$



### 5.1 Nonlinear Least-Squares

In source localization techniques, we assume that the sources are represented by a small number of point sources. The continuous lead field is effectively considered to exist only at a small set of discrete points. The forward field is typically sampled at 100 or fewer sensor locations. Inverse processing efforts are focused on (1) determining the number of sources to model, and (2) the location of these sources. We extensively review this multiple dipole model in [2], and present here a brief summary of some of the inverse techniques that can be used.

The gain matrix  $G$  is  $m \times n$  in size, where  $m$  is the number of sensor sites, and  $n$  is twice (MEG) or three times (EEG) the number of assumed dipoles. The key distinction of  $G$  here from  $G$  used in imaging discussed below is that  $m$  is assumed greater than  $n$ , preferably much greater. In either case, we denote the singular value decomposition (SVD) of  $G$  as  $U_g \Sigma_g V_g^T$ , where we retain only the principal components in  $U_g$  and  $V_g$  that correspond to non-zero singular values. We will also assume that a spatiotemporal matrix of measured data,  $F$ , has been acquired, and that its SVD is denoted as  $U_f \Sigma_f V_f^T$ , where  $U_f$  and  $V_f$  contain the principal components associated with the signal subspace. The orthogonal complements of  $U_g$  and  $U_f$  are denoted as  $\tilde{U}_g$  and  $\tilde{U}_f$ , respectively. See [2] for details on these definitions and decompositions, as well as rank determinations of these matrices.

In nonlinear least-squares (NLLS) fitting, we minimize the squared error between the data and the multiple dipole model. The parameters in the model to be fit are the locations and moments of the dipoles. In [2], we show how this problem reduces to an explicit fit of the locations only, yielding the cost function

$$J_{\text{nlls}} = \|F - GQ\| = \left\| \left( I - U_g U_g^T \right) F \right\| = \left\| \tilde{U}_g^T U_f \Sigma_f \right\| \quad (26)$$

In this case, the signal subspace spanned by  $U_f$  may include the entire space, or the user may select a lower dimensional space. For a  $p$  dipole fit, the inverse procedure is to form a gain matrix  $G$  using  $p$  candidate dipole locations, decompose it to obtain  $\tilde{U}_g$ , and calculate the cost function. The lowest cost function over all possible combinations of  $p$  locations is the NLLS solution. Exhaustively searching over all possible combinations is usually impractical, so generally a directed search algorithm is used such as Nelder-Mead Simplex, conjugate gradient, or modified Levenburg-Marquardt.

### 5.2 MUSIC

One of the primary problems of NLLS centers on the directed search algorithms. Avoiding local minima is a major concern, and determination of the parameters to drive such routines can become quite subjective. In [2], we presented an alternative algorithm that effectively allows us to exhaustively search for the solution, using a one dipole model. The assumption for the multiple signal characterization (MUSIC) algorithm is that the time series associated with each dipole is linearly independent from all other dipole time series. After careful selection of the signal subspace, the MUSIC cost function to be minimized is

$$J_M = \lambda_{\min}(U_{g1}^T \tilde{U}_f \tilde{U}_f^T U_{g1}) \quad (27)$$

which is the minimum eigenvalue of the enclosed term, which is, in turn, a  $2 \times 2$  (MEG) or  $3 \times 3$  (EEG) matrix. The procedure for MUSIC is to form the one dipole gain matrix (regardless of the true number of sources), decompose it to yield  $U_{g1}$ , and find the  $p$  locations where this function is a minimum. Exhaustively testing all practical locations of a single dipole is generally feasible, allowing us to avoid the complexities of directed searches.

### 5.3 Imaging

The basic formulation of the neuroelectromagnetic imaging problem is to construct a gain matrix  $G$  from a finely sampled grid of known source locations throughout some predefined "reconstruction" region. Since the locations are known, the solution is linear in the dipole moments, and we can then present tomographic images of these moments throughout the reconstruction regions.

The number of sensor sites is on the order of 100, and  $G$  can approach on the order of thousands or tens of thousands of columns (possible dipole sites), such that  $m \ll n$ . Fitting these discrete lead fields to spatiotemporal data is greatly underdetermined, and additional constraints must be incorporated to make the solution unique. A common assumption among many imaging approaches is to assume that the current sources are restricted to a linear combination of the lead fields. This assumption is equivalent to imposing a minimum norm requirement on the solution space. Examples of this approach can be found in [3], [4], [5], [9] and their references therein. One general drawback to minimum norm reconstruction is that the restriction of the currents to a combination of the lead fields excludes many physically plausible current configurations. In signal processing terms, the basis set spanned by the lead fields is incomplete for representing all possible current patterns. We can overcome this somewhat by adding additional sensors (and therefore lead fields), but numerical instabilities arise related to the row rank of the gain matrix.

In [1], we examined minimum norm and other reconstruction techniques for this underdetermined problem; see also discussions in [3], [4], [5], [9]. In [10], we also briefly review other inverse approaches. The many variations of minimum norm use an additional weighting matrix that has the effect of altering the  $m$ -dimensional subspace spanned by the lead fields. Since  $m \ll n$ , good solutions can only be found if the weights are carefully designed using strong *a priori* or physiologically constrained information. Arbitrary weights will simply alter this very small subspace to some other span in the much larger  $n$ -dimensional space, yielding ambiguous results.

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