EEG and MEG Source Localization using Recursively Applied (RAP) MUSIC

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Abstract

The multiple signal characterization (MUSIC) algorithm locates multiple asynchronous dipolar sources from electroencephalography (EEG) and magnetoencephalography (MEG) data. A signal subspace is estimated from the data, then the algorithm scans a single dipole model through a three-dimensional head volume and computes projections onto this subspace. To locate the sources, the user must search the head volume for local peaks in the projection metric. Here we describe a novel extension of this approach which we refer to as RAP (Recursively APplied) MUSIC. This new procedure automatically extracts the locations of the sources through a recursive use of subspace projections, which uses the metric of principal correlations as a multidimensional form of correlation analysis between the model subspace and the data subspace. The dipolar orientations, a form of "diverse polarization," are easily extracted using the associated principal vectors.

1. Introduction

The problem of localizing the sources of event related scalp potentials (the electroencephalogram or EEG) and magnetic fields (the magnetoencephalogram or MEG) can be formulated in terms of finding a least squares fit of a set of current dipoles to the observed data. Inverse methods based on direct minimization of the squared error through gradient-based optimization or simplex searches often lead to improper locations of the sources due to trapping in local minima. In an attempt to overcome this problem, we examined the use of signal subspace methods that are common in the array signal processing literature (cf. [3]). The method that we used, a variant on the MUSIC algorithm introduced in [9], replaces the multiple dipole directed search with a procedure in which a single dipole is scanned through a grid confined to a three dimensional head or source volume. At each point on this grid, the forward model for a dipole at this location is projected against a signal subspace that has been computed from the EEG and/or MEG (E/MEG) data. The locations on this grid where the source model gives the best projection onto the signal subspace correspond to the dipole locations. We also show in [7] that at each location we do not need to test all possible dipole orientations, but instead can solve a generalized eigenvalue problem whose solution gives us the orientation of the dipole (the "diverse polarization" [1], [9]) which gives the best fit to the signal space for a source at that location.

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One of the major problems with the MUSIC method, and one that is addressed by the new approach described here, is how we choose the locations which give the best projection on to the signal subspace. In the absence of noise and with perfect head and sensor models, the forward model for a source at the correct location will project entirely into the signal subspace. In practice, of course, there are errors in the estimate of the signal subspace due to noise, and errors in the forward model due to approximations in our models of the head and data acquisition system.

An additional problem is that we compute the metric only at a finite set of grid points. The effect of these practical limitations is that the user is faced with the problem of searching the gridded source volume for "peaks" and deciding which of these peaks correspond to true locations. It is important to note that a local peak in this metric does not necessarily indicate the location of a source. Only when the forward model projects entirely into the signal subspace – or as close as one would expect given errors due to noise and model mismatch – can we infer that a source is at that location. The effect of this limitation is that some degree of subjective interpretation of the MUSIC "scan" is required to decide on the locations of the sources. This subjective interpretation is clearly undesirable and can also lead to the temptation to incorrectly view the MUSIC scan as an image whose intensity is proportional to the probability of a source being present at each location.

2. Background

Quasi-static approximations of Maxwell's equations govern the relationship between neural current sources and the E/MEG data that they produce. The measurements can be expressed as an explicit function of *primary current* activity; the passive volume currents are implicitly embedded in a "lead field" formula. The model should also account for the sensor characteristics of the measurement modality, such as gradiometer orientation and configuration in MEG or differential pairs in EEG. We show in [6] that these effects can be incorporated into simple transformations that modify the basic lead field kernels. The result is that our EEG or MEG measurement $f_m(\boldsymbol{r})$ at sensor location $\boldsymbol{r}$ may be expressed as

$$f_m(\boldsymbol{r}) = \int_V \boldsymbol{g}(\boldsymbol{r}, \boldsymbol{r}') \cdot \boldsymbol{j}(\boldsymbol{r}') d\boldsymbol{r}' \qquad (1)$$

where V is the volume of sources, $\boldsymbol{j}(\boldsymbol{r}')$ represents the *primary current density* at any point $\boldsymbol{r}'$ in the volume, and $\boldsymbol{g}(\boldsymbol{r}, \boldsymbol{r}')$ is commonly known as the "lead field vector" (cf. [12]). If we assume that the primary current exists only at a discrete point $\boldsymbol{r}_q$, i.e., the primary current is $\boldsymbol{j}(\boldsymbol{r}')\delta(\boldsymbol{r}' - \boldsymbol{r}_q)$, where $\delta(\boldsymbol{r}' - \boldsymbol{r}_q)$ is the Dirac delta function, then *(1)* simplifies in E/MEG to $f_m(\boldsymbol{r}) = \boldsymbol{g}(\boldsymbol{r}, \boldsymbol{r}_q) \cdot \boldsymbol{q}$ where $\boldsymbol{q}$ is the *moment* of a *current dipole* at $\boldsymbol{r}_q$.

We assume here that our source consists of p current dipole sources. We assume simultaneous recordings at m sensors for n time instances. We can express the $m \times n$ spatio-temporal data matrix as

$$\boldsymbol{F}_m = \left[\boldsymbol{G}(\boldsymbol{r}_{q1}) \ \dots \ \boldsymbol{G}(\boldsymbol{r}_{qp})\right] \boldsymbol{Q}^T . \qquad (2)$$

We refer to the $m \times 3$ matrix $\boldsymbol{G}(\boldsymbol{r}_{qi})$ as the dipole "gain matrix" (cf. [7]), that maps a dipole at $\boldsymbol{r}_{qi}$ into a set of measurements. The three columns of the gain matrix, $\boldsymbol{G}(\boldsymbol{r}_{qi})$, represents the possible *forward fields* that may be generated by the three orthogonal orientations of the i th dipole at the m sensor locations $\{\boldsymbol{r}_1, \dots, \boldsymbol{r}_m\}$. The columns of $\boldsymbol{Q}$ represent the time series associated with each of the three orthogonal components of each dipole, i.e., with each column of the gain matrix.

For the "fixed" dipole model, whose moment orientation is time invariant, we can separate the orientation of each source from the moments as [7]:

$$\boldsymbol{F}_m = \left[\boldsymbol{G}(\boldsymbol{r}_{q1}) \ \dots \ \boldsymbol{G}(\boldsymbol{r}_{qp})\right] \cdot$$

$$\begin{bmatrix} \boldsymbol{u}_{q1} & & 0 \\ & \dots & \\ 0 & & \boldsymbol{u}_{qp} \end{bmatrix} \begin{bmatrix} s_{q1}(t_1) & \dots & s_{q1}(t_n) \\ \dots & \dots & \dots \\ s_{qp}(t_1) & \dots & s_{qp}(t_n) \end{bmatrix} \qquad (3)$$

such that $\boldsymbol{q}_i(t_i) = \boldsymbol{u}_{qi} s_{qi}(t_i)$, where $\boldsymbol{u}_{qi}$ is a unit norm orientation vector. We assume that the set of p dipoles is sufficiently spatially separated such that their gain matrices are unambiguous. The dipolar time series, however, may be linearly dependent, such that the rank of time series matrix is less than p. We therefore express *(3)* as

$$\boldsymbol{F}_m = \boldsymbol{A}(\rho, \theta)\boldsymbol{S}^T \qquad (4)$$

where $\rho \equiv \{\rho_1, \dots, \rho_r\}$ represents r clusters of dipoles, with the i th cluster comprising p_i synchronous dipoles with the location parameter set $\rho_i \equiv \{\boldsymbol{r}_{q1}, \dots, \boldsymbol{r}_{qp_i}\}$. The set $\theta \equiv \{\boldsymbol{u}_1, \dots, \boldsymbol{u}_r\}$ contains the corresponding unit norm vectors found as the extension of *(3)* to p_i dipoles. The i th column of $\boldsymbol{A}(\rho, \theta)$ is therefore found as

$$\boldsymbol{a}(\rho_i, \boldsymbol{u}_i) \equiv \left[\boldsymbol{G}(\boldsymbol{r}_{q1}) \ \dots \ \boldsymbol{G}(\boldsymbol{r}_{qp_i})\right] \boldsymbol{u}_i = \boldsymbol{G}(\rho_i)\boldsymbol{u}_i \qquad (5)$$

We refer to this column vector as a "p_i-dipolar topography." Each column of $\boldsymbol{A}(\rho, \theta)$ is therefore a p_i-dipolar topography, with a corresponding time series found as the i th column of $\boldsymbol{S}$. By regrouping the parameters in this manner, both $\boldsymbol{A}(\rho, \theta)$ and $\boldsymbol{S}$ are of full column rank and equal to the rank of our noiseless data matrix $\boldsymbol{F}_m$.

3. Subspace Correlations

Under the assumptions of spatially white i.i.d. noise, we may express the expected outer product of a noise contaminated matrix $\boldsymbol{F} = \boldsymbol{F}_m + \boldsymbol{N}$ as

$$\boldsymbol{R_F} \equiv E\{\boldsymbol{F}\boldsymbol{F}^T\} = \boldsymbol{A}\boldsymbol{S}^T\boldsymbol{S}\boldsymbol{A}^T + \sum_{i=1}^{n} E(\boldsymbol{n}(t_i)\boldsymbol{n}^T(t_i)) \qquad (6)$$

$$= \Phi_s \Lambda_F \Phi_s^T + \Phi_s^{\perp} \Lambda_n \Phi_s^{\perp T} \qquad (7)$$

where $\Lambda_F \equiv \Lambda_s + n\sigma_N^2 \boldsymbol{I}$ is the $r \times r$ diagonal matrix combining both the model and noise eigenvalues, and $\Lambda_n \equiv n\sigma_N^2 \boldsymbol{I}$ is the $(m-r) \times (m-r)$ diagonal matrix of noise-only eigenvalues. Therefore an eigendecomposition of $\boldsymbol{R_F}$ yields r eigenvalues greater than $n\sigma_N^2$. The r dimensional subspace spanned by Φ_s is the same as the r dimensional subspace spanned by the columns of $\boldsymbol{A}(\rho, \theta)$. We refer to this space as the *signal subspace*. The remaining $m - r$ vectors in $\Phi_s^{\perp}$ span the *orthogonal signal subspace*, or *noise-only subspace*,

We may estimate Φ_s from an appropriate eigenanalysis of the data matrix. We can find the source parameters, and hence the dipole locations, by comparing the column space of the matrix $\boldsymbol{A}(\rho, \theta)$ to this estimated signal subspace. We use the metric of *subspace correlations* [2] to measure the fit between these two subspaces. Since the signal subspace is spanned both by the columns of $\boldsymbol{A}(\rho, \theta)$ and the eigenvectors in Φ_s, there must exist a full rank $(r \times r)$ transformation matrix $\boldsymbol{T}$ relating the two spaces. The signal subspace is estimated from the data, such that we only have an approximation,

$$\boldsymbol{A}(\rho, \theta)\boldsymbol{T} \approx \hat{\Phi}_s . \qquad (8)$$

One approach to source localization using *(8)* is the *weighted subspace fitting (WSF)* method (cf. [3], [10], [13], [14]) in which the parameters $\{\rho, \theta, \boldsymbol{T}\}$ are found by minimizing the squared error

$$\left\| \hat{\Phi}_s \boldsymbol{W} - \boldsymbol{A}(\rho, \theta)\boldsymbol{T} \right\|_F^2 \qquad (9)$$

where $\boldsymbol{W}$ is a weighting matrix designed to improve the estimator performance [14]. Here we propose an alternative procedure in which, rather than solving directly for the parameter set $\{\rho, \theta, \boldsymbol{T}\}$, we instead examine the angles between the subspaces spanned by $\boldsymbol{A}(\rho, \theta)$ and Φ_S using *subspace correlations*.

The subspace correlation function $subcorr\{\boldsymbol{A}, \hat{\Phi}_S\} = \{s_1, s_2, \ldots, s_r\}$ defined in the Appendix yields a set of r ordered scalars $1 \ge s_1 \ge \ldots \ge s_r \ge 0$; additional details can be found in [2], [5]. These scalars are equal to the cosines of the *principal angles* between pairs of *principal vectors* chosen from the two subspaces $\boldsymbol{A}$ and $\hat{\Phi}_S$, where r is the minimum of the ranks of the two subspaces. The set of r principal vectors for each of the two subspaces are orthonormal. The first pair of principal vectors are chosen using one vector from each of the two subspaces so as to minimize the angle between the two vectors. The second pair are again chosen to minimize the angle between the vectors from the two spaces, but under the constraint that the second principal vector from $\boldsymbol{A}$ must be orthogonal to the first principal vector from $\boldsymbol{A}$, and similarly for the first two principal vectors from $\hat{\Phi}_S$. The process is repeated until a set of r pairs of principal vectors have been found, along with the associated ordered correlations corresponding to the cosines of the angles between each pair.

The computation of the subspace correlation between the signal subspace Φ_S and the matrix $\boldsymbol{A}(\rho, \theta)$ provides the fundamental basis for the RAP MUSIC algorithm. The significance of the subspace correlation function is that if one subspace is entirely contained within another, then the cosines of all the principal angles will equal unity. Conversely, if the two spaces are orthogonal, the cosines of all the principal angles will equal zero. For cases between these extremes, the set of cosine values provide a measure of the similarity between the two subspaces. The MUSIC metric corresponds to computing a subspace correlation between a single topography (the “manifold”) and the estimated signal subspace.

The subspace correlations lead to a natural extension of MUSIC. We can recursively build up our source estimate by appending putative sources to the model matrix and using the minimum of the subspace correlations as a metric for adding a new source. We define the function $distance\{\boldsymbol{A}, \hat{\Phi}_S\}$ [2] as a function of the minimum subspace correlation,

$$distance\{\boldsymbol{A}, \hat{\Phi}_S\} = \sqrt{1 - s_r^2}. \qquad (10)$$

Assuming that the rank of $\boldsymbol{A}(\rho, \theta)$ is less than or equal to that of the signal subspace estimate $\hat{\Phi}_S$, the distance as defined in *(10)* will approach zero as the column space of $\boldsymbol{A}(\rho, \theta)$ matches that of $\hat{\Phi}_S$. Consequently we can determine the parameters $\{\rho, \theta\}$ of the sources that produced the estimated signal subspace as the set that jointly minimize the distance between our topographies matrix $\boldsymbol{A}(\rho, \theta)$ and our estimated signal subspace $\hat{\Phi}_S$.

For multiple dipoles, the key concept that makes subspace distance easier to use than least-squares fitting is that if $\boldsymbol{A}(\rho, \theta)$ is parallel to Φ_S, then so is each column (each topography) of $\boldsymbol{A}(\rho, \theta)$. Since $s_r = min\{subcorr\{\boldsymbol{A}, \hat{\Phi}_S\}\}$ corresponds to the linear combination of the columns of $\boldsymbol{A}$ that minimizes the subspace correlation between the two spaces, it follows that the ith column of $\boldsymbol{A}$, i.e., the ith independent topography, must have a correlation greater than or equal to this minimum subspace correlation,

$$subcorr\{\boldsymbol{a}(\rho_i, \boldsymbol{u}_i), \hat{\Phi}_S\} \ge min\, subcorr\{\boldsymbol{A}(\rho, \theta), \hat{\Phi}_S\}. \qquad (11)$$

In our model, each column of $\boldsymbol{A}$ represents an independent topography, where each topography may comprise multiple synchronous dipoles. For exemplary purposes, let us assume that each topography represents a single current dipole. Let us further assume that we have a perfect signal subspace estimate Φ_S, in which case the minimum subspace correlation will be unity for the true parameters $\{\rho, \theta\}$. From *(11)*, each of the independent topographies formed by each dipole must also have a correlation of unity with the subspace. We can therefore find the dipole parameters by searching for the p dipole locations that each have unity correlation.

Thus a search strategy for minimizing the distance between the topographies matrix and the rank r signal subspace estimate is to search for a single dipole model whose subspace correlation is maximized with respect to $\hat{\Phi}_S$. We should find r such dipole locations in our dipolar space, each yielding a correlation value of unity. This search strategy is the basis of the MUSIC algorithm that we described in [7].

Before proceeding to a brief description of MUSIC and RAP-MUSIC, we first address the problem of finding the orientation vector $\boldsymbol{u}_i$. The dipole parameters are chosen to maximize

$$subcorr\{\boldsymbol{a}(\rho_i, \boldsymbol{u}_i), \hat{\Phi}_S\} \qquad (12)$$

However, $\boldsymbol{u}_i$ simply represents a linear combination of the columns of the gain matrix $\boldsymbol{G}(\rho_i)$. We can avoid searching for the optimal orientation vector by noting that the maximum of the subspace correlation vector $subcorr\{\boldsymbol{G}(\rho_i), \hat{\Phi}_S\}$ gives us the best way of combining the columns of $\boldsymbol{G}(\rho_i)$ so that they are as close as possible to the signal subspace. We can therefore find the optimal orientation vector $\boldsymbol{u}_i$ for each candidate location ρ_i as that which maximizes the subspace correlation at that location, i.e.:

$$max\{subcorr\{\boldsymbol{G}(\rho_i), \hat{\Phi}_S\}\} \qquad (13)$$

Therefore we can find the dipole locations by solving *(13)* at each candidate dipole location, and then searching for the true locations at which this maximum correlation equals, or is sufficiently close to, unity. Once we find these locations, we can then explicitly form the corresponding best orientation (from the Appendix, set $\boldsymbol{u}_i$ to “$\boldsymbol{x}_1$” and scale to unity norm), to determine the independent topography vector $\boldsymbol{a}(\rho_i, \boldsymbol{u}_i)$.

4. RAP-MUSIC

In [7] we adapted a "diversely polarized" form of Schmidt's original multiple signal characterization (MUSIC) algorithm ([1], [9]) to the problem of multiple point dipoles. In terms of the subspace correlations discussed here, let s_1 be the principal correlation, $max\{subcorr\{\boldsymbol{G}(\boldsymbol{r}_{qi}), \hat{\Phi}_s\}\}$. The MUSIC metric in [7] is therefore $1 - s_1^2$, effectively the square of the max correlation of a dipole gain matrix with the *noise-only* subspace, the original proposal by Schmidt. As we discussed in [7], plotting the inverse of this measure makes graphical location of the peaks easier; however, since that publication we have found it more informative to plot the principal correlation, since correlation is a direct measure of how well the model fits the data

Problems with the use of MUSIC arise when there are errors in the estimate of the signal subspace and the subspace correlation is computed at only a finite set of grid points. The largest peak is usually easily located by searching over the grid for the largest correlation; however, the second and subsequent peaks must be located by means of a three-dimensional "peak-picking" routine. While locating multiple peaks in a single parameter case (as is common in much of the MUSIC array signal processing literature) is possible, we found the problem confounding in even the simplest case of single dipolar topographies, where we must search for peaks in a three-dimensional space. Graphically searching for multiple peaks in two-dipolar topographies (a six-dimensional space) is generally not practical.

The RAP-MUSIC methods overcomes this problem by recursively building up the model. We assume that our independent topographies each comprise one or more dipoles. We search first for the single dipolar topographies, then the two-dipolar topographies, and so forth. As we discover each topography model, we add it to our existing model and continue the search. We build the source model by recursively applying the subspace correlation measure, the key metric of MUSIC, to successive subspace correlations.

For exemplary purposes, we assume that the r independent topographies each comprise a single dipole. Conceptually, RAP-MUSIC begins by finding the first dipole location to maximize *(13)*. Single dipole locations are readily found by scanning the head volume. At each point in the volume, we calculate

$$\{s_1, s_2, \dots\} = subcorr\{\boldsymbol{G}(\boldsymbol{r}_q), \hat{\Phi}_s\} \tag{14}$$

where $\{s_1, s_2, \dots\}$ is the set of subspace correlations. We find the dipole location $\hat{\boldsymbol{r}}_{q1}$ which maximizes the primary correlation s_1, then refine this location using a directed search algorithm. As discussed in the Appendix, the corresponding dipole orientation $\hat{\boldsymbol{u}}_1$ is easily obtained from the $subcorr\{\boldsymbol{G}(\hat{\boldsymbol{r}}_{q1}), \hat{\Phi}_s\}$ routine, and we designate our topography model comprising this first dipole as

$$\hat{\boldsymbol{A}}^{(1)} = \boldsymbol{a}(\hat{\boldsymbol{r}}_{q1}, \hat{\boldsymbol{u}}_1). \tag{15}$$

To search for the second dipole, we again search the head volume; however, at each point in the head, we first form the model matrix $\boldsymbol{M} = [\hat{\boldsymbol{A}}^{(1)}, \boldsymbol{G}(\hat{\boldsymbol{r}}_q)]$. We then calculate

$$\{s_1, s_2, \dots\} = subcorr\{\boldsymbol{M}, \hat{\Phi}_s\} \tag{16}$$

but now we find the dipole point that maximizes the *second* subspace correlation, s_2; the first subspace correlation should already account for $\boldsymbol{a}(\hat{\boldsymbol{r}}_{q1}, \hat{\boldsymbol{u}}_1)$ in the model. Again, an unconstrained directed search may be used to refine this second location, since the metric does not peak at the first solution. The corresponding dipole orientation $\hat{\boldsymbol{u}}_2$ may be readily obtained by projecting this second topography against the subspace, $subcorr\{\boldsymbol{G}(\hat{\boldsymbol{r}}_{q2}), \hat{\Phi}_s\}$, and we append this to our model to form

$$\hat{\boldsymbol{A}}^{(2)} = [\boldsymbol{a}(\hat{\boldsymbol{r}}_{q1}, \hat{\boldsymbol{u}}_1), \boldsymbol{a}(\hat{\boldsymbol{r}}_{q2}, \hat{\boldsymbol{u}}_2)]. \tag{17}$$

We repeat the process r times, maximizing the kth subspace correlation at the kth pass, $k = 1, \dots, r$. The final iteration is effectively attempting to minimize the subspace distance between the full r topographies matrix and the signal subspace estimate.

If the r topographies comprise r_1 single-dipolar topographies and r_2 2-dipolar topographies, then RAP-MUSIC will first extract the r_1 single dipolar models. At the $r_1 + 1$ iteration, we will find no single dipole location that correlates well with the subspace. We then increase the number of dipole elements per topography to two. We must now search simultaneously for two dipole locations, such that

$$\{s_1, s_2, \dots\} = subcorr\left\{\left[\hat{\boldsymbol{A}}^{(r_1)}, \boldsymbol{G}(\rho)\right], \hat{\Phi}_s\right\} \tag{18}$$

is maximized for the subspace correlation s_{r_1+1}, where $\rho = \{\boldsymbol{r}_{q1}, \boldsymbol{r}_{q2}\}$ comprises two dipoles. If the combinatorics are not impractical, we can exhaustively form all pairs on our grid and compute maximum subspace correlations for each pair. The alternative is to begin a two-dipole nonlinear search with random initialization points to maximize this correlation. This low-order dipole model can be easily performed using standard minimization methods.

We proceed in this manner to build the remaining r_2 2-dipolar topographies. As each pair of two dipoles is found to maximize the appropriate subspace correlation, the corresponding pair of dipole orientations may be readily obtained from $subcorr\{\boldsymbol{G}(\rho), \hat{\Phi}_s\}$, as described in the Appendix. Extensions to more dipoles per independent

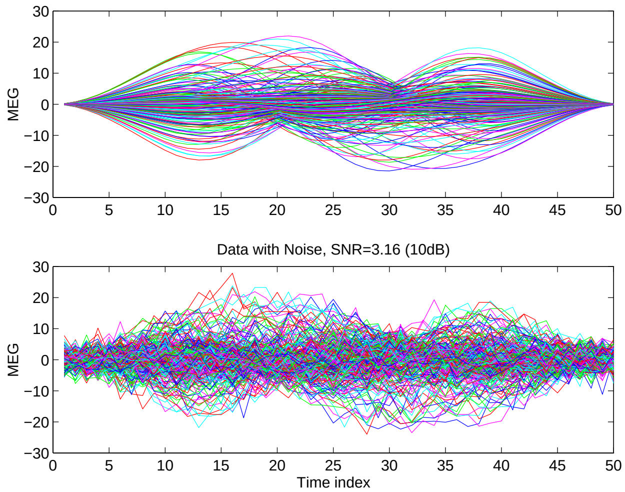

Fig. 1: Simulated MEG data, for 229 sensors by 50 time slices. True sources are three asynchronous fixed dipoles. Gaussian white noise was added such that the squared Frobenius norm of the noiseless data matrix was 3.16 times that of the noise-only matrix, i.e. 10 dB SNR

topography are straightforward, although the complexity of the search obviously increases. In any event, the complexity of the search will always remain less than the least-squares search required for finding all dipoles simultaneously.

Once we find the optimal $\{\hat{\rho}, \hat{\theta}\}$, we may find the remaining linear temporal parameters as

$$\boldsymbol{S}^T = \boldsymbol{A}^{\dagger}(\hat{\rho}, \hat{\theta})\boldsymbol{F} \tag{19}$$

i.e., we use the pseudoinverse of $\boldsymbol{A}(\hat{\rho}, \hat{\theta})$, as discussed in [7] and its references.

5. Example

We illustrate the ability of RAP-MUSIC to extract multiple correlation peaks using simulated MEG data for dipoles in a spherical head; see [6], [7] for forward model specifics. We arranged 229 radially-oriented sensors about 2 cm apart on the upper hemisphere of a 12 cm virtual sphere. Each sensor was modeled as a first-order gradiometer with a baseline separation of 5 cm. For exemplary purposes, we arranged three sources in the same z-plane, $z = 7$ cm. We fixed the orientation of each source and assigned each an independent time series. We then added white Gaussian i.i.d. noise on each sensor channel. The noiseless and noisy data are displayed in Fig. 1.

An SVD of the noisy spatio-temporal data matrix clearly showed the signal subspace to be rank three; however, to illustrate insensitivity to rank overselection, we chose a signal subspace of rank five. We created a 1.5 mm spaced grid in the correct z-plane and computed the three-dimensional gain matrix $\boldsymbol{G}(\boldsymbol{r}_{ai})$ for each location on the grid. We then computed the standard MUSIC metric *(14)* of the correlations between each gain matrix and the rank five signal subspace. As discussed in [5], our preference is to view the maximum correlations $\{s_1\}$ directly as an image; however, for publication purposes here we resort to the conventional MUSIC display of plotting $1/(1-s_1^2)$, as displayed in Fig. 2. Note that in this figure there are three peaks corresponding to the correct dipole locations and a fourth peak which represents an incorrect location. This fourth peak corresponds approximately to a dipole location that would give a local minimum in a least squares search. Since the intensity $1/(1-s_1^2)$ corresponding to this incorrect source location exceeds that of the third true source location, a MUSIC scan which picks out the three largest peaks would mislocate one of the dipoles. As we will see below, RAP MUSIC avoids this problem. We located the maximum correlation using a directed search and obtained the dipolar orientation at this point to form the first spatial topography *(15)*. We then concatenated this topography with each grid point and reran the subspace correlation of the combined model *(16)*. In Fig. 3 we see the MUSIC scan of the *second* subspace correlation, and we observe that the first source is now suppressed in the metric.

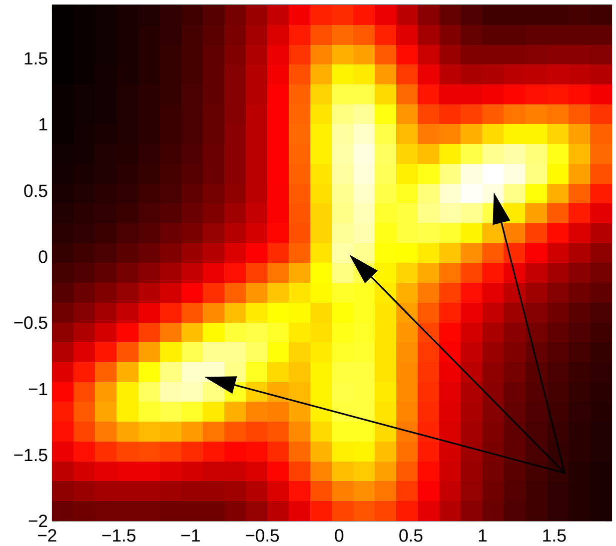

Fig. 2: MUSIC scan of dipolar models, imaging the principal correlation between model and signal subspace. The arrows indicate the true locations. The noise and partial ambiguities of dipolar models makes discerning each peak difficult. The maximum correlation was located in the grid, then used to initiate a directed search for a refined maximum of 99.1% correlation at the correct solution of $[1.0, 0.5, 7.0]$ **cm (rounded to 1 mm).**

Since this first peak is suppressed, we readily perform an unconstrained directed-search for the maximum of the

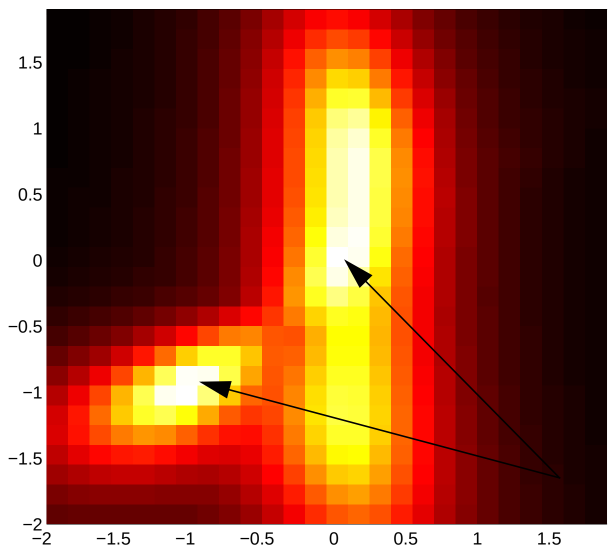

Fig. 3: MUSIC scan of *second* principal correlation, holding constant the first source from Fig. 2. The MUSIC peak from the first source is suppressed, and we readily performed a directed-search for the maximum of this second correlation, 99.1% at the correct solution of [0.0, 0.0, 7.0] **.**

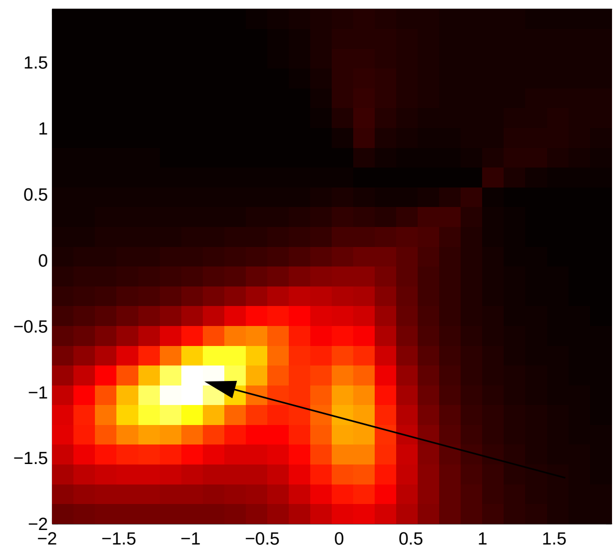

Fig. 4: MUSIC scan of *third* principal correlation, holding constant the first two sources. The MUSIC peaks from the first two sources are suppressed, and we readily performed a directed-search for the maximum of this third correlation, 99.1% at the correct solution of [–1.0, –1.0, 7.0] **.**

second subspace correlation. With the second source located, we again extracted its orientation and formed the two source topography *(17)*. We then repeated the correlation analysis to yield the MUSIC image of the *third* subspace correlation, displayed in Fig. 4. Again, a directed-search algorithm readily locates the peak of this metric. The other smaller peak in this image is spurious; after fixing the third source, a search for a fourth source yielded a maximum correlation of 26% and the algorithm was properly terminated.

6. Conclusions

RAP-MUSIC is an extension of the MUSIC algorithm for E/MEG source localization that overcomes some of the problems encountered using the basic MUSIC method described in [7]. Problems with the use of MUSIC arise when there are errors in the estimate of the signal subspace and the subspace correlation is computed at only a finite set of grid points. Locating sources requires a three-dimensional "peak-picking" routine. Suppose that an incorrect set of locations are picked. While individually each of the dipoles may have good correlations with the signal subspace, there is no guarantee that their combined source model has a small distance from the signal subspace, since we test only one dimension at a time. The RAP-MUSIC methods overcomes this problem by recursively building up the source estimate and comparing this full model to the signal subspace. By modifying our definition of the source matrix we are also able to locate synchronous sources using the RAP-MUSIC algorithm. In [5], we describe this new E/MEG spatio-temporal model, which we refer to as spatially independent topographies (SPITs), that allows direct application of RAP-MUSIC to fixed, "rotating" and synchronous dipolar sources.

By maximizing each successive subspace correlation, the RAP-MUSIC approach solves the multiple peak search problem, since each peak corresponds to a separate correlation value. We also solved a second, more subtle issue regarding this search. In a subsequent review of the signal processing literature for similar approaches, we found two comparable MUSIC algorithms, S-MUSIC [8] and IES-MUSIC [11], with the latter introduced as an extension of the former. These "successive" MUSIC algorithms were detailed for two single-parameter independent sources. The possibility of extending the successive approach to more sources in a manner similar to RAP-MUSIC is mentioned, but specifically not pursued ([11], Remark 2). Both methods, however, implement the successive search in a projection matrix approach different from the subspace correlations approach of RAP-MUSIC. As pointed out in [11], both techniques require that the search for the second source algorithmically avoid the first location. By recursively shifting to the next subspace correlation, the RAP-MUSIC algorithm bypasses this problem of previous solution points and simply maximizes each subsequent correlation.

Appendix: Subspace Correlation

Given two matrices, $\boldsymbol{A}$ and $\boldsymbol{B}$, where $\boldsymbol{A}$ is $m \times p$, and $\boldsymbol{B}$ is $m \times q$, let r be the minimum of the ranks of the two matrices. The steps to compute the subspace correlations are as follows [2] (p. 585),

1. Perform a singular value decomposition (SVD) of $\boldsymbol{A}$, such that $\boldsymbol{A} = \boldsymbol{U}_A \Sigma_A \boldsymbol{V}_A^T$. Similarly decompose $\boldsymbol{B}$. Retain only those components that correspond to non-zero singular values.
2. Form $\boldsymbol{C} = \boldsymbol{U}_A^T \boldsymbol{U}_B$. Compute the singular value decomposition, $\boldsymbol{C} = \boldsymbol{U}_C \Sigma_C \boldsymbol{V}_C^T$. Form the sets principal vectors $\boldsymbol{U_a} = \boldsymbol{U}_A \boldsymbol{U}_C$ and $\boldsymbol{U_b} = \boldsymbol{U}_B \boldsymbol{V}_C$ for sets $\boldsymbol{A}$ and $\boldsymbol{B}$ respectively.

The matrices $\boldsymbol{U_a}$ and $\boldsymbol{U}_b$ are each orthogonal, and the columns comprise the ordered set of *principal vectors* for matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ respectively. The r ordered singular values $1 \geq s_1 \geq \ldots \geq s_r \geq 0$ are extracted from the diagonal of Σ_C. The angles $\cos\theta_k = s_k$ are the *principal angles*, representing the geometric angle between the principle vectors, or analogously, s_k is the *correlation* between these two vectors. If both matrices are of the same subspace dimension, the measure $\sqrt{1 - s_r^2} = \sin\theta_r$ is called the *distance* between spaces $\boldsymbol{A}$ and $\boldsymbol{B}$ [2].

We may also readily compute the specific linear combinations of $\boldsymbol{A}$ and $\boldsymbol{B}$ that yielded these principal vectors and angles. By construction, we know that $\boldsymbol{AX} = \boldsymbol{U_a}$ for some $\boldsymbol{X}$, and $\boldsymbol{X}$ is simply found as using the pseudoinverse of $\boldsymbol{A}$: $\boldsymbol{X} = \boldsymbol{V}_A \Sigma_A^{-1} \boldsymbol{U}_C$. Similarly, we compute $\boldsymbol{Y} = \boldsymbol{V}_B \Sigma_B^{-1} \boldsymbol{V}_C$.

In E/MEG MUSIC processing, we may compute the subspace correlations between a dipole model and the signal subspace, e.g., $subcorr\{\boldsymbol{G}(\boldsymbol{r_q}), \Phi_s\}$. In this case, the orientations in $\boldsymbol{X}$ represent the dipole orientations. By scaling the first orientation to unity, $\boldsymbol{u}_1 \equiv \boldsymbol{x}_1 / \|\boldsymbol{x}_1\|$, we obtain the unit dipole orientation that best correlates the dipolar source at $\boldsymbol{r_q}$ with the signal subspace. For a two-dipolar topography, $subcorr\{[\boldsymbol{G}(\boldsymbol{r_{q1}}), \boldsymbol{G}(\boldsymbol{r_{q2}})], \Phi_s\}$, then $\boldsymbol{u}_1$ represents the concatenation of the two dipole orientations, $\boldsymbol{u}_1^T = [\boldsymbol{q}_1^T, \boldsymbol{q}_2^T]$, such that the two-dipolar topography

$$[\boldsymbol{G}(\boldsymbol{r_{q1}}), \boldsymbol{G}(\boldsymbol{r_{q2}})]\boldsymbol{u}_1 = \boldsymbol{G}(\boldsymbol{r_{q1}})\boldsymbol{q}_1 + \boldsymbol{G}(\boldsymbol{r_{q2}})\boldsymbol{q}_2 \qquad (20)$$

best correlates with the signal subspace. See [4], [5] for further discussions on subspace correlations and examples of applying them to the problem of E/MEG head modeling.

7. References

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